CBCS SCHEME

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21MAT21

Second Semester B.E./B.Tech. Degree Examination, June/July 2025
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Evaluate :
$$\int_0^2 \int_0^6 \int_0^{4-x^2} dz\ dy\ dx$$ **(06 Marks)**

b. Change the order of integration and hence evaluate $\int_0^1 \int_{\sqrt{x}}^1 (1+y)\,dy\,dx$ **(07 Marks)**

c. Define Beta and Gama function. Prove that $\beta(\frac{1}{2}, \frac{1}{2}) = \pi$ **(07 Marks)**

OR

2 a. Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, using double integration. **(06 Marks)**

b. Derive the relation between Beta and Gama function. **(07 Marks)**

c. Find the volume of the tetrahedron in the first octant bounded by $4x + 2y + z = 8$, using double integration. **(07 Marks)**

Module-2

3 a. A fluid motion is given by $\vec{v} = (y\sin z - \sin x)\hat{i} + (x\sin z + 2yz)\hat{j} + (xy\cos z + y^2)\hat{k}$ show that the motion is irrotational. **(06 Marks)**

b. Find div($\vec{v}$) and curl($\vec{v}$) of $\vec{v} = (xyz)\hat{i} + (3x^2y)\hat{j} + (xz^2 - y^2z)\hat{k}$ at $(2, -1, 1)$ **(07 Marks)**

c. Find the directional derivative of the function $\phi = x^2 - y^2 + 2z^2$ at the point P(1, 2, 3) in the direction of the vector $4\hat{i} - 2\hat{j} + \hat{k}$ **(07 Marks)**

OR

4 a. If a force $\vec{F} = 2x^2y\hat{i} + 3xy\hat{j}$ displays a particle in the xy-plane from (0, 0) to (1, 4) along the curve $y = 4x^2$. Find the work done. **(06 Marks)**

b. Using Green's theorem, evaluate $\int_C (xy - x^2)dx + x^2y\,dy$ where C is bounded by y = 0, x = 1 and y = x. **(07 Marks)**

c. Evaluate $\int_C \vec{F} \cdot \vec{dr}$ by Stoke's theorem, where $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ and C is the boundary of the rectangle $x = \pm a$, $y = 0$ and $y = b$. **(07 Marks)**

Module-3

5 a. Form partial differential equation by eliminating the arbitrary function from $z = f(x^2 + y^2)$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial y} - 6z = 0$, given that $z = x$ and $\frac{\partial z}{\partial y} = 0$, when $y = 0$. **(07 Marks)**

c. With usual notations derive a one-dimensional heat equation. **(07 Marks)**

OR

6 a. Solve : $z = yq - xp$ **(06 Marks)**

b. Form the partial differential equation from
$Z = f(y + 2x) + g(y - 3x)$ **(07 Marks)**

c. Solve : $\frac{\partial^2 z}{\partial x \partial y} = x^2 y$ subject to the conditions $z(x, 0) = x^2$ and $z(1, y) = \cos y$. **(07 Marks)**

Module-4

7 a. Find a real toot of $x \tan x = -1$ in (2.5, 3) by Regula-Falsi method in four iterations. **(06 Marks)**

b. Apply Newton's general interpolation formula to find u_x. Given that $u_0 = 8$, $u_1 = 11$, $u_4 = 68$, $u_5 = 123$. **(07 Marks)**

c. Evaluate $\int_0^3 x^4\, dx$ by Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule. (take $n = 6$). **(07 Marks)**

OR

8 a. Using Newton-Raphson method, find real root of $4x - e^x = 0$ which is near to 2. **(06 Marks)**

b. Using Simpson's $\left(\frac{3}{8}\right)^{th}$ rule, evaluate $\int_0^{0.3} \sqrt{1 - 8x^3}\, dx$ $(n = 6)$ **(07 Marks)**

c. In the following table, values of y are consecutive terms of a series of which 23.6 is the 6th term. Find the first and tenth terms of the series:

x	3	4	5	6	7	8	9
y	4.8	8.4	14.5	23.6	36.2	52.8	73.9

(07 Marks)

Module-5

9 a. Employ Taylor's series method, to find y(0.1) from $\frac{dy}{dx} = y + e^{2x}$ with $y(0) = 1$ upto 3rd degree term. **(06 Marks)**

b. Apply the Runge-Kutta method of 4th order find y at $x = 0.1$. Given that $\frac{dy}{dx} = y + x^3$; $y(0) = 1$, $(h = 0.1)$ **(07 Marks)**

c. Find y(0.4) using Milne's predictor-corrector method, given $y' = \frac{(1+x^2)y^2}{2}$; y(0) = 1, y(0.1) = 1.06, y(0.2) = 1.12, y(0.3) = 1.21 **(07 Marks)**

OR

10 a. Use Tayler's series method to find y(0.1).
Given $\frac{dy}{dx} = y + \sin x$: y(0) = 1 up to the term containing x^3. **(06 Marks)**

b. Using Modified Euler's method find y(0.1), taking h = 0.05, given that
$\frac{dy}{dx} = x^2 + y$: y(0) = 1 **(07 Marks)**

c. Given $\frac{dy}{dx} = \frac{y-x}{y+x}$: y(0) = 1, find y(0.2) taking h = 0.2, using Runge Kutta 4th order method. **(07 Marks)**

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CBCS SCHEME

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BMATC201

Second Semester B.E/B.Tech. Degree Examination, June/July 2025
Mathematics – II for Civil Engineering Stream

Time: 3 hrs. Max. Marks:100

Note: *1. Answer any FIVE full questions, choosing ONE full question from each module.*
2. M : Marks , L: Bloom's level , C: Course outcomes.
3. VTU Formula Hand Book is permitted.

			M	L	C
		Module – 1			
1	a.	Evaluate $\int_0^4 \int_0^{2\sqrt{z}} \int_0^{\sqrt{4z-x^2}} dy\,dx\,dz$.	7	L3	CO1
	b.	By changing order of integration evaluate $\int_0^a \int_0^{2\sqrt{ax}} x^2 dy\,dx$.	7	L3	CO1
	c.	Define beta and gamma functions. Show that $\sqrt{\frac{1}{2}} = \sqrt{\pi}$.	6	L2	CO1
		OR			
2	a.	Evaluate : $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\,dy$ by changing into polar coordinator.	7	L3	CO1
	b.	Find the area bounded between parabolas $y^2 = 4ax$ and $x^2 = 4ay$ using double integration.	7	L3	CO1
	c.	Write a modern mathematical program to evaluate the integral $\int_0^3 \int_0^{3-x} \int_0^{3-x-y} xyz\,dz\,dy\,dx$.	6	L3	CO5
		Module – 2			
3	a.	Find the directional derivative at $\phi = 4xz^3 - 2x^2y^2z$ at (2, –1, 2) along the vector 2i – 3j + 6k.	7	L2	CO2
	b.	If $\phi = x^2 + y^2 + z^2$ and $\vec{F} = \nabla\phi$ then find grad ϕ, div $\vec{F}$ and curl $\vec{F}$.	7	L2	CO2
	c.	Show that $\vec{F} = \frac{x_i + y_j}{x^2 + y^2}$ is both Solenoidal and irrotational.	6	L2	CO2

		OR			
4	a.	Compute the line integral $\int_c [(x^2+xy)dx+(x^2+y^2)dy]$ where c is the square formed by the lines $y=\pm 1$ and $x=\pm 1$.	**7**	**L2**	**CO2**
	b.	Apply stokes theorem to evaluate $\int_c (ydx+zdy+xdz)$ where c is the curve of intersection $x^2+y^2+z^2=a^2$ and $x+z=a$.	**7**	**L3**	**CO2**
	c.	Write a modern mathematical tool program to find the gradient of $\phi = x^2y+2xz-4$.	**6**	**L3**	**CO5**
		Module – 3			
5	a.	Form the partial differential equation by eliminating the arbitrary constant from the relation. $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.	**7**	**L2**	**CO3**
	b.	Solve the equation $\frac{\partial^2 z}{\partial u^2}=x+y$ given that $z=y^2$ when $x=0$ and $\frac{\partial z}{\partial x}=0$ when $x=2$.	**7**	**L3**	**CO3**
	c.	Solve $y^2p-xyq=x(z-2y)$.	**6**	**L3**	**CO3**
		OR			
6	a.	Form the partial differential equation by eliminating arbitrary function from the equation $z=y^2+2f\left(\frac{1}{x}+\log y\right)$.	**7**	**L2**	**CO3**
	b.	Solve $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial z}{\partial x}+2z=0$ subject to $z=e^y$ and $\frac{\partial z}{\partial x}=0$ when $x=0$.	**7**	**L3**	**CO3**
	c.	With usual notation derive a one-dimensional heat equation.	**6**	**L2**	**CO3**
		Module – 4			
7	a.	Using the Regula – Falsi method find the fifth root of 10 assuming that the root lies between 1 and 2. Carry out three approximations.	**7**	**L3**	**CO4**
	b.	Given that : <table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>y = f(x)</td><td>1</td><td>3</td><td>7</td><td>13</td></tr></table> Find the value of y at x = 0.1, by using appropriate formula.	**7**	**L3**	**CO4**
	c.	Evaluate $\int_0^{\pi/2}\sqrt{\cos\theta}\, d\theta$ by Simpson's $\frac{1}{3}^{rd}$ rule taking 7 ordinates.	**6**	**L3**	**CO4**

			Marks	Level	CO
		OR			
8	a.	Use Newton – Raphson method to find the approximate root of the equation $e^x - 3x = 0$ that lies between 0 and 1. Perform and approximate.	**7**	**L3**	**CO4**
	b.	Using Lagranges interpolation formula find f(18) for the data : <table><tr><td>x</td><td>10</td><td>12</td><td>19</td><td>22</td></tr><tr><td>y</td><td>24</td><td>48</td><td>162</td><td>200</td></tr></table>	**7**	**L3**	**CO4**
	c.	Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ by using Simpson's $\frac{3}{8}^{th}$ rule taking four equal strips.	**6**	**L3**	**CO4**
		Module – 5			
9	a.	Use Taylor's series method solve the initial value problem $\frac{dy}{dx} = xy - 1$, $y(1) = 2$ at the point x = 1.02, consider three non-zero terms.	**7**	**L3**	**CO4**
	b.	Using fourth order Runge–Kutta method find y at x = 0.1, given that $\frac{dy}{dx} = x(1+xy), y(0) = 1$.	**7**	**L3**	**CO4**
	c.	Solve the differential equation $\frac{dy}{dx} = -xy^2$ under the initial condition y(0) = 2, by using modified Euler's method at x = 0.1. Take step size h = 0.1. Perform three modification.	**6**	**L3**	**CO4**
		OR			
10	a.	Employ Taylor's series method to find y at x = 0.1 given that $\frac{dy}{dx} - 2y = 3e^x$, y(0) = 0, consider three non-zero terms.	**7**	**L3**	**CO4**
	b.	Applying Milne's predictor – corrector method compute y at x = 0.8, for the data y(0) = 2, y(0.2) = 1.9231, y =(0.4) = 1.7241 and y(0.6) = 1.4706 to the equation $\frac{dy}{dx} = -xy^2$.	**7**	**L3**	**CO4**
	c.	Write a modern mathematical tool program to solve $\frac{dy}{dx} = 2x + y$, y(1) = z by R – K 4^{th} order method.	**6**	**L3**	**CO4**

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CBCS SCHEME

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BMATE201

Second Semester B.E./B.Tech. Degree Examination, June/July 2025
Mathematics – II for EEE stream

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. VTU Formula Hand Book is permitted.
3. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	**M**	**L**	**C**
Q.1	**a.**	Find the angle between the surface $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point (1, -2, 1)	**7**	**L3**	**CO1**
	b.	Evaluate $\text{curl}(\text{curl}\vec{F})$ and $\text{div}(\text{curl}\vec{F})$ if $\vec{F} = x^2yi + y^2zj + z^2xk$.	**7**	**L3**	**CO1**
	c.	Show that the vector $\vec{F} = \dfrac{xi + yj}{x^2 + y^2}$ is both solenoidal and irrotational.	**6**	**L2**	**CO2**
		OR			
Q.2	**a.**	Find the total work done by the force $\vec{F} = 3xyi - yj + 2zxk$ in moving a particle around the circle $x^2 + y^2 = 4$.	**7**	**L3**	**CO1**
	b.	Using Green's theorem evaluate $\oint_C (xy + y^2)dx + x^2dy$ over the region bounded by the curves $y = x$ and $y = x^2$	**7**	**L2**	**CO3**
	c.	Using modern mathematical tools, write the code to find gradient of $\phi = x^2y + 2xz - 4$.	**6**	**L2**	**CO5**
		Module – 2			
Q.3	**a.**	Define a Subspace. Show that any plane passing through the origin is a subspace of R^3.	**7**	**L1**	**CO1**
	b.	Let V be the vector space of all real valued continuous functions over R. Show that the set W of solutions of differential equations $5\dfrac{d^2y}{dx^2} - 7\dfrac{dy}{dx} + 2y = 0$ is a subspace of V.	**7**	**L2**	**CO2**
	c.	Define a Inner Product space. Consider $f(t) = 3t - 5$ and $g(t) = t^2$, the inner product $\langle f.g \rangle = \int_0^t f(t)g(t)dt$. Find $\langle f \cdot g \rangle$	**6**	**L2**	**CO2**
		OR			
Q.4	**a.**	Express the vector (3, 5, 2) as a linear combination of the vectors (1, 1, 0), (2, 3, 0), (0, 0, 1) of $V_3(R)$.	**7**	**L2**	**CO3**

	b.	State Rank-Nullity theorem. Verify the Rank-Nullity theorem for the $T: R^3 \to R^3$ defined by T(x, y, z) = (x+2y-z, y+z, x+y-2z)	7	L2	CO2
	c.	Using the modern mathematical tool, write the code to represent the reflection transformation T : $R^2 \to R^2$ and to find the image of vector (10, 0) when it is reflected about the y-axis.	6	L1	CO4
		Module – 3			
Q.5	a.	Find the Laplace transform of, (i) $e^{-3t}(2\cos 5t - 3\sin 5t)$ (ii) $\dfrac{\cos at - \cos bt}{t}$	7	L3	CO3
	b.	Given $f(t) = \begin{cases} E, & 0 < t < \frac{a}{2} \\ -E, & \frac{a}{2} < t < a \end{cases}$ where f(t+a) = f(t), show that $L[f(t)] = \dfrac{E}{S}\tanh\left(\dfrac{aS}{4}\right)$.	7	L2	CO2
	c.	Express $f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \cos 2t, & \pi < t < 2\pi \\ \cos 3t, & t > 2\pi \end{cases}$ in terms of the Heaviside unit step function and hence find L[f(t)].	6	L3	CO4
		OR			
Q.6	a.	Find $L^{-1}\left[\dfrac{2s^2+5s-4}{s^3+s^2-2s}\right]$.	7	L3	CO3
	b.	Using the convolution thorem, find the inverse Laplace Transform of $\dfrac{s}{(s^2+a^2)^2}$.	7	L3	CO4
	c.	Solve $y''' + 2y'' - y' - 2y = 0$, given y(0) = $y'(0) = 0$ and $y''(0) = 6$ by using Laplace transform method.	6	L2	CO3
		Module – 4			
Q.7	a.	By Newton-Raphson method, find the root of $x \tan x + 1 = 0$ which is near to $x = \pi$.	7	L3	CO2
	b.	Using Newton's forward difference formula find f(38), $\begin{array}{\|c\|c\|c\|c\|c\|c\|c\|} \hline x & 40 & 50 & 60 & 70 & 80 & 90 \\ \hline y & 184 & 204 & 226 & 250 & 276 & 304 \\ \hline \end{array}$	7	L3	CO3
	c.	Use Lagrange's interpolation formula to find y at x = 10, given $\begin{array}{\|c\|c\|c\|c\|c\|} \hline x & 5 & 6 & 9 & 11 \\ \hline y & 12 & 13 & 14 & 16 \\ \hline \end{array}$	6	L3	CO3

		OR			
Q.8	a.	Find a real root of the equation $x^3 - 2x - 5 = 0$ correct to three decimal places by the method of false position in (2, 3).	7	L4	CO4
	b.	Find the interpolating polynomial using Newton's dividend difference formula for the following data, x: 2, 4, 5, 6, 8, 10 y: 10, 96, 196, 350, 868, 1746	7	L3	CO4
	c.	Use Simpson's $\frac{3}{8}^{th}$ rule to obtain the approximate value of $\int_0^{0.3}(1-8x^3)^{\frac{1}{2}}dx$ by considering 3 equal intervals.	6	L2	CO2
		Module – 5			
Q.9	a.	Use Taylor's series method, find y(0.1) considering upto fourth degree term if y(x) satisfies the equation, $\frac{dy}{dx} = x - y^2$, y(0) = 1.	7	L3	CO3
	b.	Given $\frac{dy}{dx} = 3x + \frac{y}{2}$, y(0) = 1. Compute y(0.2) by taking h = 0.2, using Runge-Kutta method of fourth order.	7	L2	CO3
	c.	Apply Milne's method to compute y(1.4), correct to four decimal places given $\frac{dy}{dx} = x^2 + \frac{y}{2}$ and following the data y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.4649, y(1.3) = 2.7514	6	L2	CO2
		OR			
Q.10	a.	Using modified Euler's method to find y at x = 0.2 given $\frac{dy}{dx} = x - y^2$ and y(0) = 1 by taking step size h = 0.1.	7	L4	CO4
	b.	Using the Runge-Kutta method of fourth order find y(0.1) given that $\frac{dy}{dx} = 3e^x + 2y$, y(0) = 0 taking h = 0.1.	7	L3	CO2
	c.	Using modern mathematical tools, write the code to find the solution of $\frac{dy}{dx} = x - y^2$ at y(0.1), given that y(0) = 1 by Runge-Kutta 4th order method.	6	L2	CO3

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CBCS SCHEME

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BMATM201

Second Semester B.E./B.Tech. Degree Examination, June/July 2025

Mathematics-II for ME Stream

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.
3. VTU Formula Hand Book is permitted

		Module – 1	**M**	**L**	**C**
Q.1	**a.**	Evaluate $\int_0^1\int_x^{\sqrt{x}}(x^2+y^2)dydx$.	**7**	**L3**	**CO1**
	b.	Evaluate $\int_{-1}^{1}\int_0^z\int_{x-z}^{x+z}(x+y+z)dydxdz$.	**7**	**L3**	**CO1**
	c.	Show that $\beta(m, n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.	**6**	**L2**	**CO1**
		OR			
Q.2	**a.**	Evaluate by changing into polar coordinates. $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dxdy$.	**7**	**L3**	**CO1**
	b.	Find the area of ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, by double integration.	**7**	**L2**	**CO1**
	c.	Write a modern mathematical tool program to find the volume of the tetrahedron bounded by the planes x = 0, y = 0 and $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$.	**6**	**L3**	**CO5**
		Module – 2			
Q.3	**a.**	Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at (2, –1, 2).	**7**	**L2**	**CO2**
	b.	Define a irrotational vector. Find the constants a, b and c such that the vector $\vec{F}=(x+y+az)\hat{i}+(bx+2y-z)\hat{j}+(x+cy+2z)\hat{k}$ is irrotational.	**7**	**L2**	**CO2**
	c.	If $\vec{F}=\nabla(x^3+y^3+z^3-3xyz)$ find div $\vec{F}$ and curl $\vec{F}$.	**6**	**L2**	**CO2**
		OR			
Q.4	**a.**	Find the workdone by a force $\vec{F}=(2y-x^2)\hat{i}+6yz\hat{j}-8xz^2\hat{k}$ from the point (0, 0, 0) to the point (1, 1, 1) along the straight line joining these points.	**7**	**L2**	**CO2**
	b.	Using Green's theorem, evaluate $\int_C (xy+y^2)dx+x^2dy$, where C is the closed curve of the region bounded by y = x and $y = x^2$.	**7**	**L3**	**CO2**

	c.	Write the modern mathematical tool program to find the divergence of the vector field $\vec{F} = x^2yz\,\hat{i} + y^2zx\,\hat{j} + z^2xy\,\hat{k}$.	6	L3	CO5
Module – 3					
Q.5	a.	From the PDE by eliminating the arbitrary function from $f(x + y + z, x^2 + y^2 z^2) = 0$.	7	L2	CO3
	b.	Solve $\frac{\partial^2 z}{\partial x^2} = a^2 z$ given that when x = 0, z = 0 and $\frac{\partial z}{\partial x} = a \text{ Sin } y$.	7	L3	CO3
	c.	Derive one dimensional heat equation in the standard form as $\frac{\partial u}{\partial t} = C^2 \cdot \frac{\partial^2 u}{\partial x^2}$.	6	L2	CO3
OR					
Q.6	a.	Form the PDE by eliminating the arbitrary constants 'a' and 'b' from $(x - a)^2 + (y - b)^2 + z^2 = c^2$.	7	L2	CO3
	b.	Solve $\frac{\partial^2 z}{\partial x \partial y} = \text{Sinx Siny}$ for which $\frac{\partial z}{\partial y} = -2 \text{ Siny}$, when x = 0 and z = 0, if 'y' is an odd multiple of $\pi/2$.	7	L3	CO3
	c.	Solve : $x \cdot \frac{\partial z}{\partial x} + y \cdot \frac{\partial z}{\partial y} = z$, using Lagrange's multipliers.	6	L3	CO3
Module – 4					
Q.7	a.	Find the real root of the equation $xe^x - 2 = 0$ correct to three decimal places using the Newton – Rephson method.	7	L3	CO4
	b.	Using Newton's forward interpolation formula, find y at x = 5. x: 4, 6, 8, 10 y: 1, 3, 8, 16	7	L3	CO4
	c.	Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ by taking 7 ordinates using the Trapezoidal rule.	6	L3	CO4
OR					
Q.8	a.	Compute the real root of the equation $x \log_{10}^x - 1.2 = 0$ lies between 2 and 3 by the Regula Falsi method. Carry out four approximations.	7	L2	CO4
	b.	Using Newton's divided difference formula, evaluate f(4) from the following table : x: 0, 2, 3, 6 f(x): −4, 2, 14, 158	7	L2	CO4
	c.	Compute the value of y when x = 3. Using Lagrange's interpolation formula given x: −2, −1, 1, 2 y: −7, 2, 0, 11	6	L3	CO4

		Module – 5			
Q.9	**a.**	Use Taylor's series method to find y at x = 0.1 considering the terms upto the third degree given that $\frac{dy}{dx} = x^2 + y^2$ and y(0) = 1.	**7**	**L3**	**CO4**
	b.	Apply the Runge – Kutta method of fourth order to find an approximate value of y at x = 0.2, given that $\frac{dy}{dx} = 3x + \frac{y}{2}$ with y(0) = 1, and h = 0.2.	**7**	**L3**	**CO4**
	c.	Given that $\frac{dy}{dx} = x - y^2$ and the data y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762. Compute y at x = 0.8 by applying Milne's method.	**6**	**L3**	**CO4**
		OR			
Q.10	**a.**	Using the modified Euler's method, find y(0.1) given that $\frac{dy}{dx} = x^2 + y$ and y(0) = 1 take step size h = 0.05 and perform two approximations in each stage.	**7**	**L3**	**CO4**
	b.	Using the Runge-Kutta method of fourth order, find y(0.2) given that $\frac{dy}{dx} = \frac{y-x}{y+x}$, y(0) = 1, taking h = 0.2.	**7**	**L3**	**CO4**
	c.	Using modern mathematical tools write a program to find y when x = 2 given that $\frac{dy}{dx} = 1 + \frac{y}{x}$, y(1) = 2, taking h = 0.2 by Runge – Kutta method of 4^{th} order.	**6**	**L3**	**CO5**

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CBCS SCHEME

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Second Semester B.E./B.Tech. Degree Examination, June/July 2025
Mathematics – II for CSE Stream

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. VTU Formula Hand Book is permitted.
3. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	**M**	**L**	**C**
Q.1	**a.**	Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$	**07**	**L3**	**CO1**
	b.	Evaluate $\int_{0}^{\infty}\int_{x}^{\infty}\frac{e^{-y}}{y}\,dy\,dx$ by changing the order of integration.	**07**	**L3**	**CO1**
	c.	Show that $\beta(m,n)=\frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$	**06**	**L2**	**CO1**
		OR			
Q.2	**a.**	Evaluate $\int_{0}^{1}\int_{0}^{\sqrt{1-y^2}}(x^2+y^2)\,dy\,dx$ by changing to polar coordinates.	**07**	**L3**	**CO1**
	b.	Using double integration find the area of a plate in the form of a quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	**07**	**L3**	**CO1**
	c.	Using mathematical tools, write the code to find the volume of the tetrahedron bounded by the planes x = 0, y = 0, z = 0 and 6x + 3y + 2z = 6.	**06**	**L3**	**CO5**
		Module – 2			
Q.3	**a.**	Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point (2, –1, 2).	**07**	**L2**	**CO2**
	b.	Evaluate $\text{div}\,\vec{F}$ and $\text{curl}\,\vec{F}$ at the point (1, 2, 3), given $\vec{F}=\text{grad}(x^3y+y^3z+z^3x-x^2y^2z^2)$	**07**	**L3**	**CO2**
	c.	Express the vector $\vec{F}=2x\,\hat{i}-3y^2\,\hat{j}+zx\,\hat{k}$ in cylindrical form.	**06**	**L3**	**CO2**
		OR			
Q.4	**a.**	Find the directional derivative of $\phi(x,y,z)=x^2yz+yxz^2$ at (1, –2, 1) in the direction of the vector $2\hat{i}-\hat{j}-2\hat{k}$.	**07**	**L2**	**CO2**
	b.	Show that the vector $\vec{F}=\frac{x\,\hat{i}+y\,\hat{j}}{x^2+y^2}$ is both solenoidal and irrotational.	**07**	**L3**	**CO2**
	c.	Using mathematical tool, write the code to find the gradient of xy^3+yz^3.	**06**	**L3**	**CO5**

Module – 3

Q.5 a. Show that the set $W = \{(x, y, z) \,/\, x + y + 2z = 0\}$ of the vector space $V_3(R)$ is a subspace of $V_3(R)$. **07 | L2 | CO3**

b. Find the basis and dimension of the subspace spanned by the vectors $\{(1, -1, 0), (0, 3, 1), (1, 2, 1), (2, 4, 2)\}$ in R^3. **07 | L3 | CO3**

c. Find the matrix of the linear transformation $T : V_2(R) \to V_3(R)$ such that $T(-1, 1) = (-1, 0, 2)$ and $T(2, 1) = (1, 2, 1)$. **06 | L2 | CO3**

OR

Q.6 a. Determine whether the following set of vectors in 2×2 matrix space is linearly independent or linearly dependent:

$$S = \{v_1, v_2, v_3,\} \text{ where } v_1 = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix},\ v_2 = \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} \text{ and } v_3 = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}$$

07 | L2 | CO3

b. Prove that $T : R^3 \to R^2$ defined by $T(x, y, z) = (x + y, y + 2z)$ is a linear transformation. **07 | L2 | CO3**

c. Verify the Rank – nullity theorem for the linear transformation $T : R^3 \to R^2$ defined by $T(x, y, z) = (y - x, y - z)$. **06 | L2 | CO3**

Module – 4

Q.7 a. Find a real root of the equation $x \log_{10} x = 1.2$ by regula falsi method corrected to 4 decimal places between (2.5, 3). Carry out 4 iterations. **07 | L2 | CO4**

b. From the following table of half yearly premium for policies maturing at different ages, estimate the premium for the policies maturing at the age of 46. **07 | L3 | CO4**

Age :	45	50	55	60	65
Premium (in Rs.)	114.84	96.16	83.32	74.48	68.48

c. Use Trapezoidal rule to estimate the integration

$$\int_0^1 \frac{1}{1+x^2}\,dx \quad \text{by taking 10 equal intervals.}$$

06 | L3 | CO4

OR

Q.8 a. Find by Newton's Raphson method, the root of the equation $\cos x = x e^x$ near to 0.5, corrected to 4 decimal places. **07 | L2 | CO4**

b. Using Newton's divided difference interpolation, find the interpolating polynomial of the given data: **07 | L3 | CO4**

x :	- 4	- 1	0	2	5
f(x) :	1245	33	5	9	+1335

c. Evaluate using Simpson's $(1/3)^{rd}$ rule,

$$\int_4^{5.2} \log x\,dx \quad \text{by dividing the range (4, 5.2) into seven ordinates.}$$

06 | L3 | CO4

Module – 5

Q.9	a.	Solve $y' = 3x + y^2$; $y(0) = 1$ using Taylor's series method at $x = 0.1$ and $x = 0.2$.	07	L3	CO4
	b.	Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with $y = 1$ when $x = 0$. Find an approximate y at $x = 0.1$ using Euler's modified method. (Use modified formula twice).	07	L3	CO4
	c.	From the data given below find y at $x = 1.4$ using Milne's method. Given $\frac{dy}{dx} = x^2 + \frac{y}{2}$. x : 1, 1.1, 1.2, 1.3 y : 2, 2.2156, 2.4549, 2.7514	06	L3	CO4
		OR			
Q.10	a.	Using Runge – Kutta method of order 4, find y at $x = 0.2$, given $\frac{dy}{dx} = \frac{x^2 + y^2}{10}$; $y(0) = 1$ taking $h = 0.1$.	07	L3	CO4
	b.	Using modified Euler's method solve $y' = 3x + \frac{y}{2}$ with $y(0) = 1$ taking $h = 0.2$ at $x = 0.2$. (Use modified Euler's formula twice).	07	L3	CO4
	c.	Using mathematical tools, write the code to solve $\frac{dy}{dx} = x^2y - 1$; $y(0) = 1$ by Taylor's series method at $x = 0.1\ (0.1)\ 0.3$	06	L3	CO5

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