CBCS SCHEME

USN | | | | | | | | | | **21MAT31**

Third Semester B.E./B.Tech. Degree Examination, June/July 2025
Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the Laplace transform of

$$t^5 e^{4t}\cosh 3t + 5^t + \frac{\sin 2t}{t}$$ **(06 Marks)**

b. Express $f(t) = \begin{cases} \cos t & \text{for} \quad 0 < t \le \pi \\ 1 & \text{for} \quad \pi < t \le 2\pi \\ \sin t & \text{for} \quad \pi > 2\pi \end{cases}$ in terms of unit step function and hence find its Laplace transform. **(07 Marks)**

c. Using convolution theorem find the inverse Laplace transform of $\dfrac{1}{s^3(s^2+1)}$. **(07 Marks)**

OR

2 a. Find the inverse Laplace transform of

$$\frac{3s+2}{s^2-s-2}$$ **(06 Marks)**

b. If $f(t) = \begin{cases} t & \text{for} \quad 0 \le t \le \pi \\ 2\pi - t & \text{for} \quad \pi \le t \le 2\pi \end{cases}$ and $f(t+2\pi) = f(t)$ then

show that $L\{f(t)\} = \dfrac{1}{s^2}\tanh\left(\dfrac{\pi s}{2}\right)$ **(07 Marks)**

c. Use Laplace transform to solve $y'' + 6y' + 9y = 12t^2\, e^{-3t}$ under the conditions $y(0) = y'(0) = 0$. **(07 Marks)**

Module-2

3 a. Expand

$$f(x) = \frac{\pi - x}{2} \quad \text{in } 0 \le x \le 2\pi$$ as Fourier series expansion. **(06 Marks)**

b. Obtain the Fourier series for the function

$$f(x) = \begin{cases} 1 + \dfrac{4x}{3} & \text{in} \quad -\dfrac{3}{2} < x < 0 \\ 1 - \dfrac{4x}{3} & \text{in} \quad 0 < x < \dfrac{3}{2} \end{cases},$$

hence deduce that $\dfrac{\pi^2}{8} = \dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \ldots\ldots\ldots$ **(07 Marks)**

c. Expand $f(x) = \sin x$ in Fourier half range cosine series over the interval $(0, \pi)$. **(07 Marks)**

OR

4 a. Find the Fourier series expansion of x^2 in $-\pi \le x \le \pi$. **(06 Marks)**

b. Obtain the Fourier half range sine series for the function

$$f(x) = \begin{cases} \frac{1}{4} - x & \text{in} \quad 0 < x < \frac{1}{2} \\ x - \frac{3}{4} & \text{in} \quad \frac{1}{2} < x < 1 \end{cases}$$ **(07 Marks)**

c. A function f(x) of period 2π is specified by the following table.

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
f(x)	7.9	7.2	3.6	0.5	0.9	6.8	7.9

Obtain the Fourier series for f(x) upto the first harmonic. **(07 Marks)**

Module-3

5 a. Find the Fourier transform of

$$f(x) = \begin{cases} 1 & \text{in} \quad |x| \le 1 \\ 0 & \text{in} \quad |x| > 1 \end{cases}$$ and hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. **(06 Marks)**

b. Find the Fourier sine and cosine transform of $f(x) = e^{-ax}$ for $a > 0$ and $x > 0$. **(07 Marks)**

c. Obtain the inverse z-transform of $\frac{3z^2 + z}{(5z-1)(5z+2)}$ **(07 Marks)**

OR

6 a. Find the Fourier transform of the function

$$f(x) = \begin{cases} 1 - |x| & \text{for} \quad |x| \le 1 \\ 0 & \text{for} \quad |x| > 1 \end{cases}$$, hence evaluate $\int_0^\infty \frac{\sin^2 x}{x^2} dx$. **(06 Marks)**

b. Find the z-transform of (i) $\cos n\theta$ (ii) $\sin(n\theta)$ **(07 Marks)**

c. Solve $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$ with $u_0 = u_1 = 0$ using z-transform. **(07 Marks)**

Module-4

7 a. Classify the second order partial differential equations :

(i) $u_{xx} + 2u_{xy} + u_{yy} = 0$ (ii) $(x+1)u_{xx} - 2(x+2)u_{xy} + (3+x)u_{yy} = 0$

(iii) $y^2u_{xx} + u_{yy} + u_x^2 + u_y^2 + 7 = 0$ (iv) $(1+x^2)u_{xx} + (5+2x^2)u_{xt} + (4+x^2)u_{tt} = 0$ **(10 Marks)**

b. Solve the elliptic equation $u_{xx} + u_{yy} = 0$ [Laplace equation] for the following square mesh with boundary values as shown in Fig.Q7(b). Use Leibamann's method for 1st iteration.

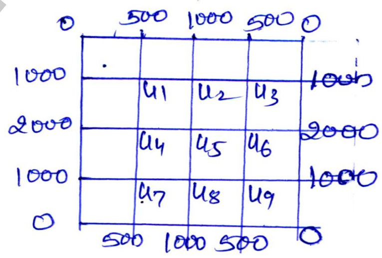

Fig.Q7(b) **(10 Marks)**

OR

8 a. Solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2},$$ subject to $u(0, t) = u(4, t) = 0$, $\frac{\partial u}{\partial t}(x, 0) = 0$ and $u(x, 0) = x(4 - x)$ by taking step length in x, h = 1. **(10 Marks)**

b. Solve $u_t = u_{xx}$ subject to the conditions $u(0, t) = u(1, t) = 0$, $u(x, 0) = \sin(\pi x)$, $0 \le t \le 0.1$ by taking h = 0.2, by applying Bendre-Schemidt explicit formula, hence find
(i) u(0.2, 0.04) (ii) u(0.6, 0.06) **(10 Marks)**

Module-5

9 a. By fourth order Runge-Kutta method, solve

$$\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$$ for x = 0.2, correct to four decimal places using the initial conditions y = 1 and $\frac{dy}{dx} = 0$, when x = 0. **(06 Marks)**

b. Derive Euler's equation in the standard form

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$$ **(07 Marks)**

c. Show that the equation of the curve joining the points (0, 1) and (1, 2) for which $I = \int_0^1 \sqrt{1 + (y')^2}\, dx$ is extremum, is a straight line. **(07 Marks)**

OR

10 a. Apply Milne's method to find y(0.4) from the $y'' + xy' + y = 0$ and initial values as y(0) = 1, y(0.1) = 0.995, y(0.2) = 0.9801, y(0.3) = 0.956, $y'(0) = 0$, $y'(0.1) = -0.0995$, $y'(0.2) = -0.196$, $y'(0.3) = -0.2867$. **(07 Marks)**

b. Prove that geodesics on a plane are straight line. **(06 Marks)**

c. Find the curve on which the functional $\int_0^{\pi/2} \left[(y')^2 - y^2 + 2xy\right] dx$ with $y(0) = y(\pi/2) = 0$ can be extremised. **(07 Marks)**

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CBCS SCHEME

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BMATEC301/BEC301/BBM301

Third Semester B.E./B.Tech. Degree Examination, June/July 2025
AV Mathematics III for EC/ BM Engineering

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.
3. Use of Statistical table and Formula hand book is permitted.

		Module – 1	**M**	**L**	**C**
Q.1	**a.**	Find the Fourier Series expansion of the function $f(x)=\lvert x\rvert$ in $(-\pi,\pi)$.	**06**	**L2**	**CO1**
	b.	Find the half range Fourier sine series for function, $f(x)=\begin{cases}\frac{1}{4}-x & \text{in } 0<x<\frac{1}{2}\\ x-\frac{3}{4} & \text{in } \frac{1}{2}<x<1\end{cases}$	**07**	**L2**	**CO1**
	c.	Find the constant term and the first co-efficients of cosine and sine terms in the Fourier series expansion for the following data : $\begin{array}{\|c\|c\|c\|c\|c\|c\|c\|}\hline x & 0 & 1 & 2 & 3 & 4 & 5\\ \hline y & 9 & 18 & 24 & 28 & 26 & 20\\ \hline\end{array}$	**07**	**L3**	**CO1**
		OR			
Q.2	**a.**	Obtain the Fourier series of the square wave given by, $f(x)=\begin{cases}-K & \text{in } -\pi<x<0\\ K & \text{in } 0<x<\pi\end{cases}$	**06**	**L2**	**CO1**
	b.	Obtain the half range cosine series for $f(x)=(x-1)^2$ in $0\le x\le 1$.	**07**	**L2**	**CO1**
	c.	Obtain the constant term and coefficients of first cosine and sine terms in the expansion of 'y' from the following table : $\begin{array}{\|c\|c\|c\|c\|c\|c\|c\|}\hline x^\circ & 0 & 60^\circ & 120^\circ & 180^\circ & 240^\circ & 300^\circ\\ \hline y & 7.9 & 7.2 & 3.6 & 0.5 & 0.9 & 6.8\\ \hline\end{array}$	**07**	**L3**	**CO1**
		Module – 2			
Q.3	**a.**	Find the Fourier transform of f(x) = $\begin{cases}1-\lvert x\rvert, & \lvert x\rvert\le a\\ 0, & \lvert x\rvert>a\end{cases}$. Hence evaluate $\int_0^\infty \frac{\sin^2 t}{t^2}\,dt$.	**06**	**L3**	**CO2**
	b.	Find the Fourier cosine transform of the function $f(x)=e^{-\alpha x}$. Evaluate $\int_0^\infty \frac{\cos mx}{\alpha^2+x^2}\,dx$	**07**	**L2**	**CO2**
	c.	Find the DFT of a sequence $x(n)=\{1,1,0,0\}$ and also find the IDFT of $Y(K)=\{1,0,1,0\}$.	**07**	**L3**	**CO2**
		OR			

Q.4	**a.**	Find the Fourier transform of $f(x)=\begin{cases}1-x^2, & \lvert x\rvert<1\\ 0, & \lvert x\rvert\ge 1\end{cases}$	**06**	**L2**	**CO2**
	b.	Find the Fourier cosine transform of $f(x)=\begin{cases}4x, & \text{for } 0<x<1\\ 4-x, & \text{for } 1<x<4\\ 0, & \text{for } x>4\end{cases}$	**07**	**L1**	**CO2**
	c.	Find the Fourier sine transform of $f(x)=e^{-\lvert x\rvert}$ and hence evaluate $\int_0^{\infty}\frac{x\sin mx}{1+x^2}dx,\ m>0.$	**07**	**L3**	**CO2**
		Module – 3			
Q.5	**a.**	Find the z-transform of : (i) $(n+1)^2$ (ii) $\sinh n\theta$.	**06**	**L2**	**CO3**
	b.	Find the inverse z-transform of $\frac{z}{(z-1)(z-2)}$.	**07**	**L3**	**CO3**
	c.	Solve the difference equation of $y_{n+2}-4y_n=0$, given that $y_0=0$ and $y_1=2$ using Z-transform.	**07**	**L3**	**CO3**
		OR			
Q.6	**a.**	Find the z-transform of $\sin(3n+5)$.	**06**	**L2**	**CO3**
	b.	Obtain the inverse z-transform of $\frac{2z^2+3z}{(z+2)(z-4)}$.	**07**	**L3**	**CO3**
	c.	If $U(z)=\frac{2z^2+3z+4}{(z-3)^3}$, evaluate u_2 and u_3.	**07**	**L3**	**CO3**
		Module – 4			
Q.7	**a.**	Solve $(D^3-3D^2+3D-1)y=0$.	**06**	**L2**	**CO4**
	b.	Solve $\frac{d^2y}{dx^2}-4y=\cosh(2x-1)+3^x$	**07**	**L2**	**CO4**
	c.	Solve $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+2y=\sin(\log x)$	**07**	**L3**	**CO4**
		OR			
Q.8	**a.**	Solve $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=3\sin x$.	**06**	**L2**	**CO4**
	b.	Solve $(3x+2)^2y''+3(3x+2)y'-36y=4(3x+2)^2$	**07**	**L2**	**CO4**
	c.	In an LCR circuit the charge 'q' on a plate of a condenser is given by, $L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{c}=E\sin pt$. Solve the above equation.	**07**	**L3**	**CO4**
		Module – 5			
Q.9	**a.**	Find a least square straight line for the following data : $\begin{array}{\|c\|c\|c\|c\|c\|c\|}\hline x & 5 & 10 & 15 & 20 & 25\\ \hline y & 16 & 19 & 23 & 26 & 30\\ \hline\end{array}$	**06**	**L2**	**CO5**
	b.	Compute $\bar{x}$, $\bar{y}$ and r from the following equation of the regression lines $2x+3y+1=0$; $x+6y-4=0$.	**07**	**L3**	**CO5**

	c.	Ten students got the following percentage of marks in two subjects say x and y, compute their rank correlation co-efficient.	**07**	**L3**	**CO5**

x	78	36	98	25	75	82	90	62	65	39
y	84	51	91	60	68	62	86	58	53	47

OR

Q.10	**a.**	Fit a parabola $y = ax^2 + bx + c$ by the method of least squares for the following data :	**06**	**L2**	**CO5**

x	2	4	6	8	10
y	3.07	12.85	31.47	57.38	91.29

	b.	Find the correlation co-efficient between x and y for the following data :	**07**	**L3**	**CO5**

x	1	2	3	4	5
y	2	5	3	8	7

	c.	Find the two regression lines from the following data :	**07**	**L3**	**CO5**

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

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