CBCS SCHEME

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21MAT41

Fourth Semester B.E./B.Tech. Degree Examination, June/July 2025

Complex Analysis, Probability and Statistical Methods

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Define analytic function and derive Cauchy–Riemann equations in polar form. **(06 Marks)**

b. If f(z) is analytic, show that :

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]|f(z)|^2 = 4|f'(z)|^2.$$ **(07 Marks)**

c. Evaluate $\int_c \frac{e^{2z}}{(z+1)(z+2)}dz$, where c is the circle $|z| = 3$. **(07 Marks)**

OR

2 a. Show that $f(z) = z + e^z$ is analytic, and hence find its derivative interms of z. **(06 Marks)**

b. Find the analytic function f(z) = u + iv, given that $u = x^2 - y^2 + \frac{x}{x^2+y^2}$ by Milne Thomson method. **(07 Marks)**

c. State and prove Cauchy's integral formula. **(07 Marks)**

Module-2

3 a. Show that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$. **(06 Marks)**

b. Obtain the series solution of Bessel's differential equation :

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-n^2)y=0.$$ **(07 Marks)**

c. Express $x^3 - 5x^2 + x + 2$ in terms of Legendre polynomials. **(07 Marks)**

OR

4 a. Obtain the series solution of Legendre differential equation :

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0.$$ **(06 Marks)**

b. Express x^3+2x^2-4x+5 in terms of Legendre polynomial. **(07 Marks)**

c. Prove that $J_{\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\sin x$. **(07 Marks)**

Module-3

5 a. Find Karl Pearson's coefficient of correlation for the following data :

x	10	14	18	22	26	30
y	18	12	24	6	30	36

(06 Marks)

b. Fit a straight line y = ax + b for the data :

x	5	10	15	20	25
y	16	19	23	26	30

(07 Marks)

c. Obtain the lines of regression and hence find the coefficient of correlation for the data :

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(07 Marks)

OR

6 a. Ten students got the following percentage of marks in two subjects x and y. Compute their rank correlation coefficient.

Marks in x	78	36	98	25	75	82	90	62	65	39
Marks in y	84	51	91	60	68	62	86	58	53	47

(06 Marks)

b. Compute the means $\bar{x}, \bar{y}$ and the correlation coefficient r from the given regression lines,
4x – 5y + 33 = 0
20x – 9y = 107. **(07 Marks)**

c. Fit a Parabola $y=ax^2+bx+c$ by the method of least squares for the following data :

x	2	4	6	8	10
y	3.07	12.85	31.47	57.38	91.29

(07 Marks)

Module-4

7 a. A random variable X has the following probability function :

X	0	1	2	3	4	5	6
P(X)	K	3K	5K	7K	9K	11K	13K

Find K.
Also find :
i) $P(x \geq 5)$
ii) $P(3 < x \leq 6)$. **(06 Marks)**

b. Derive the mean and variance of Poisson distribution. **(07 Marks)**

c. The probability that a pen manufactured by a factory be defective is $\frac{1}{10}$. If 12 such pens are manufactured, what is the probability that :
i) Exactly two are defective
ii) Atleast two are defective
iii) None of them are defective. **(07 Marks)**

OR

8 a. A random variable X has the density function :

$$f(x) = \begin{cases} kx^2 & -3 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Evaluate K.
Also find :
i) $P(1 \leq x \leq 2)$
ii) $P(x \leq 2)$
iii) $P(x > 1)$. **(06 Marks)**

b. The probability that an individual suffers a bad reaction from as injection is 0.001. Find the probability that out of 2000 individuals.
i) Exactly three
ii) More than 2 will get bad reaction. **(07 Marks)**

c. In a normal distribution 31% of the items are under 45 and 8% over 64. Find the mean and S.D. Given A(0.5) = 0.19 and A(1.4) = 0.42. **(07 Marks)**

Module-5

9 a. The joint probability distributions of two random variables are given below :

X \ Y	-4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

Determine :
i) Marginal distributions of x and y
ii) E[X] and E[Y]
iii) Verify X and Y are independent. **(06 Marks)**

b. Define :
i) Null hypothesis
ii) Type – I and Type – II errors
iii) Level of significance. **(07 Marks)**

c. A certain stimules administered to each of 12 patients resulted in the following changes in the blood pressure, 5, 2, 8, –1, 3, 0, 6, –2, 1, 5, 0, 4. Can it be concluded that stimules will increase the blood pressure [$t_{0.05}$ for 11 d.f is 2.201]. **(07 Marks)**

OR

10 a. The joint probability distribution of two random variables X and Y are given below :

X \ Y	-3	2	4
1	0.1	0.2	0.2
2	0.3	0.1	0.1

Determine :
i) E(X) and E(Y)
ii) E[X Y]
iii) COV(X, Y). **(06 Marks)**

b. Find the student 't' test for the following variable values in a sample of 8 are – 4, –2, –2, 0, 2, 2, 3, 3. Taking the mean of the universe to be zero. **(07 Marks)**

c. The theory predicts the proportion of beans in the four group A, B, C and D should be 9 : 3 : 3 : 1. In an experiment among 1600 beans, the number in the four groups were 882, 313, 287 and 118. The goodness of fit χ^2 values of above data is approximately equal to? $\left(\chi^2_{0.05} = 5.99\right)$. **(07 Marks)**

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CBCS SCHEME

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21MATCS41

Fourth Semester B.E./B.Tech. Degree Examination, June/July 2025
Mathematical Foundations for Computing, Probability and Statistics

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define tautology. Show that .
$\neg(p \to q) \to \neg q$ is a tautology by constructing truth table. **(06 Marks)**

b. Suppose the Universe consists of all integers. Consider the following open statements :
$P(x) : x \leq 3$, $q(x) = x+1$ is odd, $r(x) : x > 0$
Write down the truth values of the following statements :
(i) $q(1)$
(ii) $\neg p(3)$
(iii) $p(3) \wedge q(4)$
(iv) $p(7) \vee q(7)$
(v) $\neg p(3) \vee r(0)$
(vi) $p(0) \to q(0)$
(vii) $\neg[p(-4) \vee q(-3)]$ **(07 Marks)**

c. Give an indirect proof and proof by contradiction for the statement "If n^2 is an odd then n is odd." **(07 Marks)**

OR

2 a. Prove the following using laws of logic,
$[\neg p(\neg q \wedge r)] \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$. **(07 Marks)**

b. Test the validity of the following argument using Rules of Inference
If Ravi goes out with friends, he will not study
If Ravi does not study, his father becomes angry
His father is not angry

$\therefore$ Ravi has not gone out with friends **(07 Marks)**

c. Define (i) Open statements (ii) Quantifiers **(06 Marks)**

Module-2

3 a. Consider the function $f : R \to R$ defined by $f(x) = 2x+5$. Let a function $g : R \to R$ be defined by $g(x) = \frac{1}{2}(x-5)$. Prove that g is an inverse of f. **(06 Marks)**

b. Relation R defined on the set A = {1, 2, 3, 4} is shown in the following matrix.

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Write down R. Verify that R is an equivalence relation and determine the partition induced. **(07 Marks)**

c. In the following figure, determine (i) a path from b to d (ii) a walk from b to d that is not a trail (iii) a closed walk from b to b that is not a circuit (iv) a circuit from b to b that is not a cycle. **(07 Marks)**

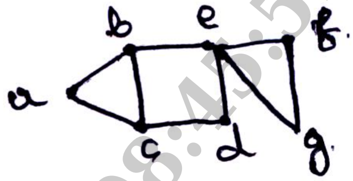

Fig. Q3 (c)

OR

4 a. Let $f : A \rightarrow B$, $g : B \rightarrow C$, are any two functions. Prove that,

(i) If $g \circ f : A \rightarrow C$ is onto, then g is onto.

(ii) If $g \circ f : A \rightarrow C$ is one-to-one, then f is one-to-one. **(06 Marks)**

b. Let $A = \{1,2,3,4,6,12\}$. Define a relation R on A by aRb if and only if "b is a multiple of a". Write down the relation R. Prove that R is a Poset and draw the Hasse diagram for this relation. **(07 Marks)**

c. Define graph isomorphism. Determine whether the following graphs are isomorphic or not. **(07 Marks)**

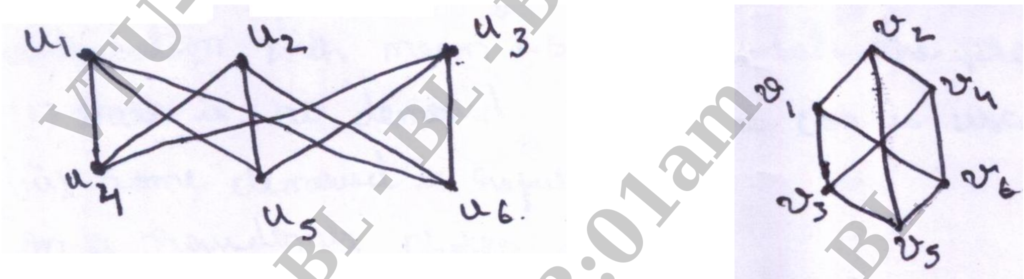

Fig. Q4 (c)

Module-3

5 a. If P is the pull required to lift a load W by means of a pulley block, find a law in the form of a straight line $P = mW + C$ by using the following data. Compute P when W = 150 kg wt.

P	12	15	21	25
W	50	70	100	120

(06 Marks)

b. Find the correlation co-efficient between x and y :

x	1	2	3	4	5	6	7	8	9	10
y	10	12	16	28	25	36	41	49	40	50

(07 Marks)

c. The regression equations of z variables x and y are $x = 0.7y + 5.2$ and $y = 0.3x + 2.8$. Find the means $\bar{x}$ and $\bar{y}$. Also find the coefficient of correlation between x and y. **(07 Marks)**

OR

6 a. Following table gives the data on rainfall and discharge in a certain river. Obtain the line of regression of y on x.

Rainfall –x (inch)	1.53	1.78	2.60	2.95	3.42
Discharge y (1000 cc)	33.5	36.3	40.0	45.8	53.5

What is the estimated discharge corresponding to rainfall of 3 inch. **(06 Marks)**

b. Find the rank correlation for the following data :

x	78	36	98	25	75	82	90	62	65	39
y	84	51	91	60	68	62	86	58	53	47

(07 Marks)

c. Fit a parabola $y = a + bx + cx^2$ to the following data :

x	2	4	6	8	10
y	3.07	12.85	31.47	57.38	91.29

(07 Marks)

Module-4

7 a. A random variable X has the following probability function:

x	1	2	3	4	5	6
p(x)	0.2	K	0.1	2K	0.3	3K

Find K and calculate mean and standard deviation of X. **(06 Marks)**

b. A car hire firm has 2 cars which it hires out day by day. The number of demand for a car is distributed as a Poisson distribution with mean 1.5. Calculate the probability that
- (i) There is no demand
- (ii) One car is used.
- (iii) Some demand is refused.

On a randomly choosen day. **(07 Marks)**

c. In a normal distribution, 7% are under 35 and 11% are over 63. Find the mean and standard deviation. Given A(1.48) = 0.43 and A(1.23) = 0.39 in the usual notation. **(07 Marks)**

OR

8 a. A function is defined as, $P(x) = \begin{cases} \frac{1}{18}(2x+3), & \text{if } 2 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$. Show that P(x) is a density function. Find mean of x. **(06 Marks)**

b. The probability that a bomb dropped from a plane will strike the target is $\frac{1}{5}$. If 6 bombs are dropped, find the probability that, (i) Exactly 2 will strike the target (ii) At least 1 will strike the target. **(07 Marks)**

c. The life of a certain type of electrical lamp is normally distributed with a mean of 2040 hrs and standard deviation of 60 hrs. In a consignment of 2000 lamps, find how many would be expected to burn for, (i) more than 2150 hrs (ii) less than 1950 hrs (iii) between 1920 hrs and 2160 hrs. Given A(1.83) = 0.4664, A(1.5) = 0.4332, A(2) = 0.4772. **(07 Marks)**

Module-5

9 a. Explain the terms : (i) Sample (ii) Null hypothesis (iii) Type I and Type II error **(06 Marks)**

b. A sample height of 6400 soldiers has a mean of 172.34 cms and a standard deviation of 6.5 cms, while a sample of height of 1600 soldiers has a mean of 174.12 cms and a standard deviation of 6.4 cms. Does the data indicate that sailors are on the average taller than soldiers? Use the left tailed test at 0.01 level of significance. Given $-Z_C = -2.33$ **(07 Marks)**

c. The following table gives the number of road accidents that occurred in a city during the various days of a week. Test the hypothesis that the accidents are uniformly distributed over all the days of the week. Given $\chi^2_{0.01}(6) = 16.81$

Day	Sun	Mon	Tue	Wed	Thu	Fri	Sat
No. of accidents	14	16	8	12	11	9	14

(07 Marks)

OR

10 a. Find the joint distributions of X and Y, which are independent random variables with the following respective distributions and find mean X, Y and XY. **(06 Marks)**

X	1	2
P(X)	0.7	0.3

Y	-2	5	8
P(Y)	0.3	0.5	0.2

b. A die was thrown 1200 times and the number 6 was obtained 236 times. Can the die be considered fair at 0.01 level of significance? ($Z_C = 2.58$). **(07 Marks)**

c. A random sample of 10 students have the following I.Q.
70, 120, 110, 101, 88, 83, 95, 98, 107, 100
Does this data support the hypothesis that the population mean I.Q. is 100 at 5% level of significance? Given that $t_{0.05}(g) = 2.26$. **(07 Marks)**

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21MATME41

Fourth Semester B.E./B.Tech. Degree Examination, June/July 2025
Complex Analysis, Probability and Linear Programming

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. Statistical tables permitted.

Module-1

1 a. Show that the function $f(z) = |z|^2$ is differentiable but not analytic at the origin. **(06 Marks)**

b. Find the analytic function f(z) = u + iv. Given $u - v = \dfrac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}$ and $f\left(\dfrac{\pi}{2}\right) = 0$. **(07 Marks)**

c. If f(z) is analytic function of Z, prove that $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. **(07 Marks)**

OR

2 a. With usual notations, derive the Cauchy-Riemann equations in polar form. **(06 Marks)**

b. Show that W = logz is analytic every were except at z = 0 and hence find its derivative. **(07 Marks)**

c. Find an analytic function f(z) = u + iv whose imaginary part is $v = r^2 \cos 2\theta - r\cos\theta + 2$. **(07 Marks)**

Module-2

3 a. Discuss the transformation $w = z + \dfrac{1}{z}$. **(06 Marks)**

b. Evaluate the integral $\int_c |z|^2 dz$, were c is the square having vertices at the origin 0 and the points P(1, 0), Q(1, 1) and R(0, 1). **(07 Marks)**

c. Find the bilinear transformation that transforms the points $z_1 = i$, $z_2 = 1$, $z_3 = -1$ onto the points $w_1 = 1$, $w_2 = 0$, $w_3 = \infty$ respectively. **(07 Marks)**

OR

4 a. If c is the simple closed curve in the complex plane, evaluate the following :

i) $\int_c \dfrac{dz}{z-a}$ ii) $\int_c \dfrac{dz}{(z-a)n}$, n = 2,3,..... on the cases a is outside c and a is inside c. **(06 Marks)**

b. Using the Cauchy's integral formula evaluate $\int_c \dfrac{z}{z^2 - 3z + 2} dz$ were c is the circle i) $|z-2| = \dfrac{1}{2}$ and ii) $|z| = 1$. **(07 Marks)**

c. Discuss the transformation $w = z^2$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-3

5 a. The probability distributive function P(X) of a variate x is given by the following table :

X	0	1	2	3	4	5	6
P(X)	K	3K	5K	7K	9K	11K	13K

For what value of K, does this represent a valid probability distribution? Find $P(x < 4)$, $P(x \geq 5)$ and $P(3 < x \leq 6)$. **(06 Marks)**

b. The probability that a man aged 60 will live to be 70 is 0.65. What is the probability that out of 10 men, now aged 60,
i) Exactly 9 will live to be 70
ii) At most 9 will live to be 70
iii) At least 7 will live to be 70? **(07 Marks)**

c. If x is an exponential variate with mean 4, evaluate the following :
i) $P(0 < x < 1)$ ii) $P(x > 2)$ iii) $P(-\infty < x < 10)$. **(07 Marks)**

OR

6 a. A random variable x has the density function $P(x) = \begin{cases} kx^2, & 0 \leq x \leq 3 \\ 0, & \text{elsewere} \end{cases}$, evaluate k and find
i) $P(x \leq 1)$ ii) $P(1 \leq x \leq 2)$ iii) $P(x > 1)$ **(06 Marks)**

b. A certain screw making machine has a chance of producing 2 defective out of 1000. The screws are packed in boxes of 100. Using poison distribution, find the approximate number of boxes containing i) no defective screw ii) two defective screws in a consignment of 5000 boxes. **(07 Marks)**

c. In a normal distribution 7% are under 35 and 89% are over 63. Find the mean and standard deviation, given that A(1.23) = 0.39 and A(1.48) = 0.43, in the usual notation. **(07 Marks)**

Module-4

7 a. Use the simplex method to maximize $z = 3x + 4y$ subject to the constraints :
$2x + y \leq 40$, $2x + 5y \leq 180$, $x \geq 0$, $y \geq 0$. **(10 Marks)**

b. Use Big-M method solve the L.P.P. minimize $z = 2x + 3y$ subject to the constraints $x + y \geq 5$, $x + 2y \geq 6$, $x \geq 0$, $y \geq 0$. **(10 Marks)**

OR

8 a. Define the following terms: A linear programming problem, basic solution, basic feasible solution, optimal solution, artificial variables of an LPP. **(10 Marks)**

b. Solve the following LP problem by the using two-phase simplex method.
Minimize $z = x - 2y - 3z$
Subject to the constraints $-2x + y + 3z = 2$,
$2x + 3y + 4z = 1$ and
$x, y, z \geq 0$ **(10 Marks)**

Module-5

9 a. Determine an initial basic feasible solution to the following transportation problem by using i) Least cost method ii) North west corner method.

	D_1	D_2	D_3	D_4	Supply
S_1	21	16	15	3	11
S_2	17	18	14	23	13
S_3	32	27	18	41	19
	6	10	12	15	

(10 Marks)

b. ABC company is engaged in manufacturing 5 brands of packed snacks. It is having five manufacturing setups, each capable of manufacturing any of its brands, one at a time. The cost make a brand on these setups vary according to the following table:

	S_1	S_2	S_3	S_4	S_5
B_1	4	6	7	5	11
B_2	7	3	6	9	5
B_3	8	5	4	6	9
B_4	9	12	7	11	10
B_5	7	5	9	8	11

Find assignment of brands to snacks are optimum. **(10 Marks)**

OR

10 a. Determine an initial basic feasible solution of the following transportation problem by using Vogel's approximation method:

	D_1	D_2	D_3	D_4	Supply
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demand	200	225	275	250	

(10 Marks)

b. Five men are available to do five different jobs. From past records, the time (in hours) that each man takes to do each job is known and given in the following table:

Job

	Men \ Job	I	II	III	IV	V
Men	A	2	9	2	7	1
	B	6	8	7	6	1
	C	4	6	5	3	1
	D	4	2	7	3	1
	E	5	3	9	5	1

Find assignment of men to jobs that will minimize the total time taken. **(10 Marks)**

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